CS 259

Security in Process Calculi

Vitaly Shmatikov

Overview

- Pi calculus
  - Core language for parallel programming
  - Modeling security via name scoping
- Applied pi calculus
  - Modeling cryptographic primitives with functions and equational theories
  - Equivalence-based notions of security
  - A little bit of operational semantics
  - Security as testing equivalence

Pi Calculus

[Milner et al.]

- Fundamental language for concurrent systems
  - High-level mathematical model of parallel processes
  - The "core" of concurrent programming languages
  - By comparison, lambda-calculus is the "core" of functional programming languages
- Mobility is a basic primitive
  - Basic computational step is the transfer of a communication link between two processes
  - Interconnections between processes change as they communicate
- Can be used as a general programming language

A Little Bit of History

- 1980: Calculus of Communicating Systems (CCS) [Milner]
- 1992: Pi Calculus [Milner, Parrow, Walker]
  - Ability to pass channel names between processes
- 1998: Spi Calculus [Abadi, Gordon]
  - Adds cryptographic primitives to pi calculus
  - Security modeled as scoping
  - Equivalence-based specification of security properties
  - Connection with computational models of cryptography
- 2001: Applied Pi Calculus [Abadi, Fournet]
  - Generic functions, including crypto primitives

Pi Calculus Syntax

- Terms
  - M, N ::= x *variables*
    - | n *names*
    - *Let u range over names and variables*
- Processes
  - P,Q ::= nil *empty process*
    - | $\bar{u}\langle N\rangle$.P *send term N on channel u*
    - | u(x).P *receive term from channel P and assign to x*
    - | !P *replicate process P*
    - | P|Q *run processes P and Q in parallel*
    - | (νn)P *restrict name n to process P*

Modeling Secrecy with Scoping

- A sends M to B over secure channel c

$$
\begin{aligned}
A(M) &= \bar{c}\langle M\rangle \\
B &= c(x).\mathrm{nil} \\
P(M) &= (\nu c)(A(M)\,|\,B)
\end{aligned}
$$

This restriction ensures that channel c is "invisible" to any process except A and B (other processes don't know name c)

Secrecy as Equivalence

$$
\begin{aligned}
A(M) &= \bar{c}\langle M\rangle \\
B &= c(x).nil \\
P(M) &= (\nu c)\,(A(M)\,|\,B)
\end{aligned}
$$

Without (νc), attacker could run process c(x) and tell the difference between P(M) and P(M')

- P(M) and P(M') are "equivalent" for any values of M and M'
  - No attacker can distinguish P(M) and P(M')
- Different notions of "equivalence"
  - Testing equivalence or observational congruence
  - Indistinguishability by any probabilistic polynomial-time Turing machine

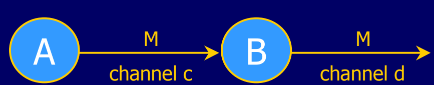
Another Formulation of Secrecy

$$
\begin{aligned}
A(M) &= \bar{c}\langle M\rangle \\
B &= c(x).nil \\
P(M) &= (\nu c)\,(A(M)\,|\,B)
\end{aligned}
$$

- No attacker can learn name n from P(n)
  - Let Q be an arbitrary attacker process, and suppose it runs in parallel with P(n)
  - For any process Q in which n does not occur free, P(n) | Q will never output n

Modeling Authentication with Scoping

- A sends M to B over secure channel c
- B announces received value on public channel d



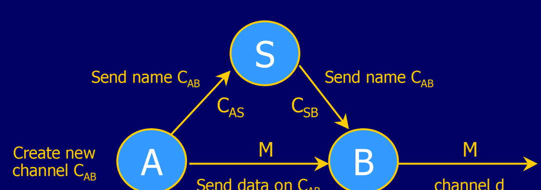
$$
\begin{aligned}
A(M) &= \bar{c}\langle M\rangle \\
B &= c(x).\bar{d}\langle x\rangle \\
P(M) &= (\nu c)\,(A(M)\,|\,B)
\end{aligned}
$$

Specifying Authentication

$$
\begin{aligned}
A(M) &= \bar{c}\langle M\rangle \\
B &= c(x).\bar{d}\langle x\rangle \\
P(M) &= (\nu c)\,(A(M)\,|\,B)
\end{aligned}
$$

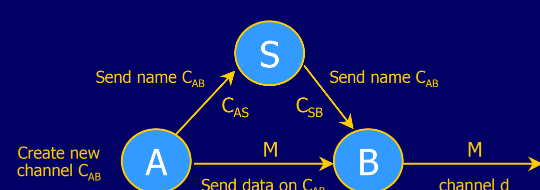
- For any value of M, if B outputs M on channel d, then A previously sent M on channel c

A Key Establishment Protocol



1. A and B have pre-established pairwise keys with server S
   - Model these keys as names of pre-existing communication channels
2. A creates a new key and sends it to S, who forwards it to B
   - Model this as creation of a new channel name
3. A sends M to B encrypted with the new key, B outputs M

Key Establishment in Pi Calculus



$$
\begin{aligned}
A(M) &= (\nu c_{AB})\,\overline{c_{AS}}\langle c_{AB}\rangle.\overline{c_{AB}}\langle M\rangle \\
S &= c_{AS}(x).\overline{c_{SB}}\langle x\rangle \\
B &= c_{SB}(x).x(y).\bar{d}\langle y\rangle \\
P(M) &= (\nu c_{AS})\,(\nu c_{SB})\,(A(M)\,|\,B\,|\,S)
\end{aligned}
$$

Note communication on a channel with a dynamically generated name

Applied Pi Calculus

- In pi calculus, channels are the only primitive
- This is enough to model some forms of security
  - Name of a communication channel can be viewed as an "encryption key" for traffic on that channel
    - A process that doesn't know the name can't access the channel
  - Channel names can be passed between processes
    - Useful for modeling key establishment protocols
- To simplify protocol specification, applied pi calculus adds functions to pi calculus
  - Crypto primitives modeled by functions and equations

Applied Pi Calculus: Terms

M, N ::=	x	*Variable*
	| n	*Name*
	| $f(M_1,\ldots,M_k)$	*Function application*

- Standard functions
  - pair(), encrypt(), hash(), …
- Simple type system for terms
  - Integer, Key, Channel⟨Integer⟩, Channel⟨Key⟩

Applied Pi Calculus: Processes

P,Q ::=	nil	*empty process*
	| ū⟨N⟩.P	*send term N on channel u*
	| u(x).P	*receive from channel P and assign to x*
	| !P	*replicate process P*
	| P|Q	*run processes P and Q in parallel*
	| (νn)P	*restrict name n to process P*
	| if M = N then P else Q	*conditional*

Modeling Crypto with Functions

- Introduce special function symbols to model cryptographic primitives
- Equational theory models cryptographic properties
- Pairing
  - Functions pair, first, second with equations:
    - first(pair(x,y)) = x
    - second(pair(x,y)) = y
- Symmetric-key encryption
  - Functions symenc, symdec with equation:
    - symdec(symenc(x,k),k)=x

More Equational Theories

- Public-key encryption
  - Functions pk,sk generate public/private key pair pk(x),sk(x) from a random seed x
  - Functions pdec,penc model encryption and decryption with equation:
    - pdec(penc(y,pk(x)),sk(x)) = y
  - Can also model "probabilistic" encryption:
    - pdec(penc(y,pk(x),z),sk(x)) = y

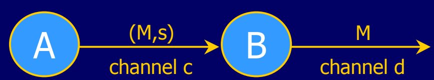
      Models random salt (necessary for semantic security)
- Hashing
  - Unary function hash with no equations
  - hash(M) models applying a one-way function to term M

Yet More Equational Theories

- Public-key digital signatures
  - As before, functions pk,sk generate public/private key pair pk(x),sk(x) from a random seed x
  - Functions sign,verify model signing and verification with equation:
    - verify(y,sign(y,sk(x)),pk(x)) = y
- XOR
  - Model self-cancellation property with equation:
    - xor(xor(x,y),y) = x
  - Can also model properties of cyclic redundancy codes:
    - crc(xor(x,y)) = xor(crc(x),crc(y))

Dynamically Generated Data

- Use built-in name generation capability of pi calculus to model creation of new keys and nonces

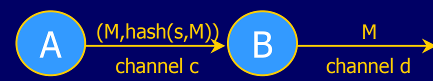



```
A(M) = c̄⟨(M,s)⟩
  B  = c(x).if second(x)=s
          then d̄⟨first(x)⟩
P(M) = (νs)(A(M)|B)
```

Models creation of fresh capability every time A and B communicate

capability s may be intercepted!

Better Protocol with Capabilities



Hashing protects integrity of M and secrecy of s

```
A(M) = c̄⟨(M,hash(s,M))⟩
  B  = c(x).if second(x)=
              hash(s,first(x))
           then d̄⟨first(x)⟩
P(M) = (νs)(A(M)|B)
```

Proving Security

- "Real" protocol
  - Process-calculus specification of the actual protocol
- "Ideal" protocol
  - Achieves the same goal as the real protocol, but is secure by design
  - Uses unrealistic mechanisms, e.g., private channels
  - Represents the desired behavior of real protocol
- To prove the real protocol secure, show that no attacker can tell the difference between the real protocol and the ideal protocol
  - Proof will depend on the model of attacker observations

Example: Challenge-Response

- Challenge-response protocol

  $A \rightarrow B \quad \{i\}_k$

  $B \rightarrow A \quad \{i+1\}_k$

- This protocol is secure if it is indistinguishable from this "ideal" protocol

  $A \rightarrow B \quad \{random_1\}_k$

  $B \rightarrow A \quad \{random_2\}_k$

Example: Authentication

- Authentication protocol

  $A \rightarrow B \quad \{i\}_k$

  $B \rightarrow A \quad \{i+1\}_k$

  $A \rightarrow B$ "Ok"

- This protocol is secure if it is indistinguishable from this "ideal" protocol

  $A \rightarrow B \quad \{random_1\}_k$

  $B \rightarrow A \quad \{random_2\}_k$

  $B \rightarrow A \quad random_1, random_2$ on a magic secure channel

  $A \rightarrow B$ "Ok" if numbers on real & magic channels match

Security as Observational Equivalence

- Need to prove that two processes are observationally equivalent to the attacker
- Complexity-theoretic model
  - Prove that two systems cannot be distinguished by any probabilistic polynomial-time adversary [Beaver '91, Goldwasser-Levin '90, Micali-Rogaway '91]
- Abstract process-calculus model
  - Cryptography is modeled by abstract functions
  - Prove testing equivalence between two processes
  - Proofs are easier, but it is nontrivial to show computational completeness [Abadi-Rogaway '00]

Structural Equivalence

$$
\begin{aligned}
P \mid nil &\equiv P \\
P \mid Q &\equiv Q \mid P \\
P \mid (Q \mid R) &\equiv (P \mid Q) \mid R \\
!P &\equiv P \mid !P \\
(\nu m)\,(\nu n)P &\equiv (\nu n)\,(\nu m)P \\
(\nu n)nil &\equiv nil \\
(\nu n)(P \mid Q) &\equiv P \mid (\nu n)Q && \textit{if n is not a free name in P} \\
P[M/x] &\equiv P[N/x] && \textit{if M=N in the equational theory}
\end{aligned}
$$

Operational Semantics

- Reduction $\rightarrow$ is the smallest relation on closed processes that is closed by structural equivalence and application of evaluation contexts such that

$$\bar{a}\langle M\rangle.P \mid a(x).Q \;\rightarrow\; P \mid Q[M/x]$$

models P sending M to Q on channel a

$$\text{if } M = M \text{ then } P \text{ else } Q \;\rightarrow\; P$$

$$\text{if } M = N \text{ then } P \text{ else } Q \;\rightarrow\; Q$$

for any ground M, N s.t. $M \neq N$ in the equational theory

Equivalence in Process Calculus

- Standard process-calculus notions of equivalence such as bisimulation are not adequate for cryptographic protocols
  - Different ciphertexts leak no information to the attacker who does not know the decryption keys
- $(\nu k)\bar{c}\langle symenc(M,k)\rangle$ and $(\nu k)\bar{c}\langle symenc(N,k)\rangle$ send different messages, but they should be treated as equivalent when proving security
  - In each case, a term is encrypted under a fresh key
  - No test by the attacker can tell these apart

Testing Equivalence

- Informally, two processes are equivalent if no environment can distinguish them
- A test is a process R and channel name w
  - Informally, R is the environment and w is the channel on which the outcome of the test is announced
- A process P passes a test (R,w) if P | R may produce an output on channel w
  - There is an interleaving of P and R that results in R being able to perform the desired test
- Two processes are equivalent if they pass the same tests

Advantages and Disadvantages

- Proving testing equivalence is hard
  - Need to quantify over all possible attacker processes and all tests they may perform
  - There are some helpful proof techniques, but no fully automated tools and very few decision procedures
- Testing equivalence is a congruence
  - Can compose protocols like building blocks
- Equivalence is the "right" notion of security
  - Direct connection with definitions of security in complexity-theoretic cryptography
  - Contrast this with invariant- and trace-based definitions

Bibliography

- Robin Milner. "Communication and Concurrency". Prentice-Hall, 1989.
  - Calculus of communicating systems (CCS)
- Robin Milner. "Communicating and Mobile Systems: the π-Calculus". Cambridge University Press, 1999.
  - Pi calculus
- Martin Abadi and Andrew Gordon. "A calculus for cryptographic protocols: the spi-calculus". Information and Computation 148(1), 1999.
  - Spi calculus
- Martin Abadi and Cedric Fournet. "Mobile values, new names, and secure communication". POPL 2001.
  - Applied pi calculus
- Martin Abadi and Phillip Rogaway. "Reconciling two views of cryptography". Journal of Cryptology 15(2), 2002.
  - On equivalence of complexity-theoretic and process-calculus models